Theory of Population Profiles and the Ukrainian Death Spiral

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Abstract

Numerical experiments are conducted on the population equation with the aim of reproducing the shapes of population profiles, or pyramids. Fertility is defined in terms of a ratio of the population of women of child-bearing years to children 0-4 years old. The profile of the child-bearing women is assumed to be described by a δ-function or a Gaussian peaking at 20-32.5 years of age, with Gaussian width $\sigma \sim 4$ years, a fertility factor f ($f<1$ for population decline and $f>1$ for population growth), and sex ratio $\rho \approx 1$. The results are applied to the Ukrainian population, now hollowed out of both men and women of peak childbearing years due to the Russia/Ukraine War. The most likely projection implies less than 20 million Ukrainians in 50 years.

> The happiness of a country does not depend absolutely upon its poverty or riches, upon its youth or its age, upon its being thinly or fully inhabited, but upon the rapidity with which it is increasing, upon the degree in which the yearly increase of food approaches to the yearly increase of an unrestricted population.
>
> --Thomas Robert Malthus, *An Essay on the Principle of Population* (1798)

Introduction

One doesn't have to completely agree with Malthus's characterization of happiness to recognize a truth at the heart of what he says: A happy country is a growing, well-fed country. A corollary of Malthus's remark might be that a country at war is an unhappy country, and especially unhappy are those who have to flee the country of their birth.

Demographic studies can quantify the happiness of a country by counting births and immigrants, and its misery by counting deaths and exiles.

Because the age- and gender-stratified population pyramids of a given country are rarely found in the shapes of pyramids (Nigeria notwithstanding), the term *population profiles* is preferred. They are the same thing. See PopulationPyramid.net[1] for almost limitless (~150x(2024-1950) ~ 11,000) examples of population profiles, as well as projections of profiles to the year 2100, all based on UN data, which is here assumed throughout to be valid.

A theory of population profiles is presented. Numerical experiments are conducted to generate evolving population profiles for different female fertility functions. In this study, the migration term in the population equation is assumed to be negligible. The birth rate depends on the fertility of young females in the ~15-40 yo range. Female fertility profiles with age are modeled by a δ-function or Gaussian functions at peak female fertility age τ_{pk}.

The framework is used to make predictions for the future of Ukraine under various scenarios. Our results bode ill for Ukraine, whose young and educated females have, in significant numbers, taken asylum in EU countries, and whose young men (other than the sons of oligarchs and the well-connected) have been sacrificed in the name of an international rules-based-order that makes the rules that befits its order.

The Russo-Ukraine war, known in Russia as the Special Military Operation, is an ongoing catastrophe for Ukraine.

Theory

A formal presentation of the population theory informing my earlier work[2,3,4] is given here, now with updated notation.

Let $N_i(\tau;t)d\tau \equiv \left[\frac{dN_i(\tau;t)}{d\tau}\right] d\tau$ represent the differential number of persons of type i with ages τ in the range τ to $\tau+d\tau$. The index i may refer to biological gender, country, epoch, or any further subdivision or grouping of persons with a census record and a tally of births and deaths over an extended period of time. Better still if the data include the birth date, nationality and age of the birth mother. Numbers or estimates of numbers or, better still, demographic data of migrants improve the data analysis further.

Here, migration is neglected and, lacking the maternal birthdate data, some assumptions will be made to characterize the female fertility profile.

Equating the total time derivative of $N_i(\tau;t)$ with population sources (births and immigration) and sinks (deaths and emigration) gives

$$\frac{dN_i(\tau;t)}{dt} = \frac{\partial N_i(\tau;t)}{\partial t} + \left|\frac{d\tau}{dt}\right| \frac{\partial N_i(\tau;t)}{\partial \tau} = B_i(t)\delta(\tau) + J_i(\tau;t) - \nu_i(\tau,t)\, N_i(\tau;t), \quad (1)$$

or

$$\frac{\partial N_i(\tau;t)}{\partial t} = -\frac{\partial N_i(\tau;t)}{\partial \tau} + B_i(t)\delta(\tau) + J_i(\tau;t) - \nu_i(\tau,t)\, N_i(\tau;t), \quad (2)$$

Here we use the relation $d\tau/dt = 1$, perhaps the saddest equation in all of physics. Though vast sums are spent to arrest the effects of aging, we the living age at precisely the same rate at which time passes. Only two things can be done to stop aging: dying or violating causality, the former inevitable, the latter impossible.

Eq.(2) can almost be written down by inspection. Only the first term on the right-hand-side, the aging function, needs a little explanation. The negative sign is because, should the population have a negative slope at some age τ and epoch *t*, that is, if it has more younger than older persons with ages near τ, then as time passes, the population near age τ will positively increase as the more numerous young age through τ.

The 2nd term on the rhs of eq.(2) is the birth function at time t, $B_i(t)$ [yr^{-1}], which employs the δ-function for zero-age birth. Some uncertainty arises in defining birth rate, as there can be significant mortality during a child's first year of life (though usually still a small fraction of the total birth rate†). Practically speaking, then, the birth rate is defined as the number of babies that reach their first birthday or in the calculations below, by relating the birth rate to the population of 0-4 yo children.

The 3rd term on the rhs of Eq.(2) is the migration function $J_i(\tau;t)$ [τ^{-1} yr^{-1}], representing time-dependent positive age-stratified immigration into the county minus age-stratified emigration out of the country. The term $J_i(\tau;t)$ is neglected In this study. Exploring the effects of migration will be the topic of a future paper; see below with respect to Ukraine, and Appendix B in Ref. [4] for a preview.

The 4th term, the mortality function, was recently studied in some detail.[2] We use the very simplest characterization that contains the essential features of human mortality. Contemporary old-age male and female mortality rates are assumed to be described by the function

$$\nu_M(\tau, t)\ [\,yr^{-1}] = \nu_{min} + \nu_o \exp(k_1\tau)\ . \quad (3)$$

Here $k_1 \equiv \ln(\nu_{\tau_2}/\nu_{\tau_1})\ /(\tau_2 - \tau_1)$, ν_{τ_1} and ν_{τ_2} are the mortality rates at τ_1 and τ_2 years of age and time *t* taken from fits to the mortality rates of the elderly, and $\nu_0 = \nu_1 \exp(-k_1\tau_1)$. The index *i* for sexual gender or country is suppressed in ν_{min} and ν_0. The background mortality ν_{min} is observed to be in the range $10^{-3} - 10^{-2}$ yr^{-1}, though it is highly country-specific and epoch-dependent and certainly not flat with age. Thus, in generality, the mortality $\nu_{imin} = \nu^i_{min}(\tau,t)$, and also $\nu^i_{\tau1,2} = \nu^i_{\tau1,2}(t)$ and $\nu^i_{\tau1,2} = \nu^i_{\tau1,2}(t)$. Future studies should consider the time-evolution of these coefficients, and a more realistic representation for the mortality of the young (<~ 60 yo).

For the present study, a time-independent mortality law of the form given by eq.(3) is assumed. The survival probability of the average person in a cohort of persons who live to age τ and suffer mortality described by eq.(3) is obtained from the steady-state solution to eq.(2) assuming a constant birth rate B, and is given by[2]

$$N(\tau) = B\ \exp\left[-\nu_{min}\tau - \left(\frac{\nu_0}{k_1}\right)\left(e^{k_1\tau} - 1\right)\right]\ . \qquad (4)$$

Here we treat the birth function as a boundary condition at $\tau \to 0$, the birth age.

Eq.(4) is a synthetic population profile for a highly contrived situation with a constant birth rate over >~100 yrs or, equivalently, eq.(4) gives the average survival probabilities of randomly chosen individuals of age τ in a population characterized by the mortality law of eq.(3).

†According to AMA data analyzed by Sandra Adamson Fryhofer,[5] the US maternal mortality rate was 20.1,23.8, and 32.9 deaths per 100,000 live births for 2019, 2020, and 2021, respectively. If a woman has, on average, 2 children, then maternal mortality is ~ 30x2/100,000/(20 yrs) ~ 3×10^{-5} yr^{-1} for ~20-40 yo US women. Maternal mortality therefore represents a small fraction of total female mortality in this age range. In comparison, the US birth rate per year in the years 2019- 2021 were around 12/1000 people,[6] representing a birth fertility of 1.2%, a factor some 30 times larger than the maternal mortality. The Ukrainian birth rate is considered below, in Fig. 4b.

Numerical Experiment

The unit of time is the yr, and the unit of rate is yr^{-1}. Calculations of the evolution of population profiles subject to time-dependent birth-rate and specific mortality and fertility functions over several hundred year time frames are carried out. The <~ 0.1% of the population who are centenarians is not modeled here. For purposes of illustration, a medium mortality law deduced from populations of US men in recent times is used.

Upon substituting eq.(3), the evolution equation (2) with no migration becomes

$$\frac{\partial N_i(\tau;t)}{\partial t} = -\frac{\partial N_i(\tau;t)}{\partial \tau} - [\, \nu^i_{min} + \nu^i_0 \exp(k_1\tau)\,]\; N_i(\tau;t)\;, \quad (5)$$

with boundary condition $N_i(\tau \to 0; t) = B_i(t)$. This equation is easily solved with the Euler method using a grid spacing for age of 0.1 yr.

First consider a hypothetical scenario of a constant birth rate that is imposed by diktat in order to guarantee a steady population. Assuming peak fertility age of women is τ_{ok} = 25 years, and a "background" female mortality of ν^F_{min}= 0.001 yr^{-1} then, on average 97.5% of women reach age 25, So the female fertility factor should be $f_{F\,=}$ 1.025 (baby girls per female). (Different notation for fertility is used in the next section.)

Sex-selective abortion can greatly alter the population profiles (see, e.g., a recent UAE population profile).

We suppose that in the test population, females give birth on average to 1.1 baby boys for every 1.025 baby girls, irrespective of maternal age. Thus the fertility factor for males is f_M = 1.1. Perhaps because of the more wanton and reckless behavior of males, or perhaps because of a worse genetic endowment, a somewhat larger value of background mortality is here assumed for men, namely, ν^M_{min}= 0.003 yr^{-1}

Again, mortality of the young is undoubtedly multi-causal and highly structured, not to mention temporally evolving. A flat mortality rate for the young is used only to reveal profile behaviors in simple, artificial cases.

For the birth rate $B_i(t)$, we use (i) a δ-function at the peak fertility age τ_{pk}, and (ii) a Gaussian peaking at τ_{pk} with a width σ(yr). In case (i),

$$B_i(t) = f_i(t) N_F(\tau_{pk}; t). \quad (6)$$

In case (ii),

$$B_i(t) = f_i(t)\, \frac{1}{\sqrt{2\pi}\sigma} \int_{\tau_y}^{\tau_l} d\tau \exp\left[-\frac{(\tau-\tau_{pk})^2}{2\sigma^2}\right]\; N_F(\tau;t). \quad (7)$$

Only girls older than τ_y = 10 yo and women younger than τ_l = 50 yo are assumed to give birth. The results for the considered scenarios are not, however, sensitive to somewhat different lower and upper maternal age limits.

We examine the three fertility scenarios listed in Table 1 below with method (ii), eq.(7), recognizing that realistic fertility functions require empirical fits to countrywide data for the mother's age and the child's birth date and sexual gender. It is also understood that these guesses may reflect the author's social biases. A fuller empirical study is needed for each country under consideration (some of which must surely be available in the subscription literature). In any case, a 2-sided Gaussian seems a next obvious generalization

Table 1. Gaussian fertility coefficients for the birth rate, eq. (7)

	t_{pk} (yr)	σ (yr)
Early	22.5	3
Middle	25	4 or 5
Late	32.5	5

The early fertility scenario has peak female fertility peaking at τ_{ok} = 22.5 years of age, and a narrow Gaussian width σ = 3 years. Most of child-rearing is performed in this scenario before women reach 30 years of age. The late fertility scenario, by contrast, has τ_{ok} = 32.5 year, and a broader width of 5 years. A middle scenario, with τ_{pk} = 25 yo and σ = 4 years, is used in the calculations unless specified otherwise.

For illustrative calculations, we adopt the mortality law for the US elderly derived in our recent studies[2,4]. In Ref. [4], US average male mortalities were used to estimate the life expectancy and survival times for US men the same age as our current and former presidents, using US average male morality. Here we use input coefficients for the medium mortality of US men: $\tau_1 = 60, \tau_2 = 100,\ \nu_1^M = 0.012, \nu_2^M = 0.56$., and of US women: $\tau_1 = 65, \tau_2 = 100,\ \nu_1^F = 0.013;\ \nu_2^F = 0.46$; cf. Fig.1 of Ref.[4].

Results

Fig. 1 shows the evolution of the population profile with time. For the first hundred years, the profile reaches a steady state described by eq.(4), which is by construction with *f* =1 and b_F = 1.025. Then, for the next 100 years, *f* is set equal to a different value, in this case, 0.5. Fig. 1 shows the evolution of the profile as a result of this fertility collapse. The population at 200 years is only a small, ~15%, fraction of the population at 100 years.

To derive the birth rate in the calculations, an integration over the age-stratified female population is performed using a Gaussian for the female fertility function, eq. (7), with

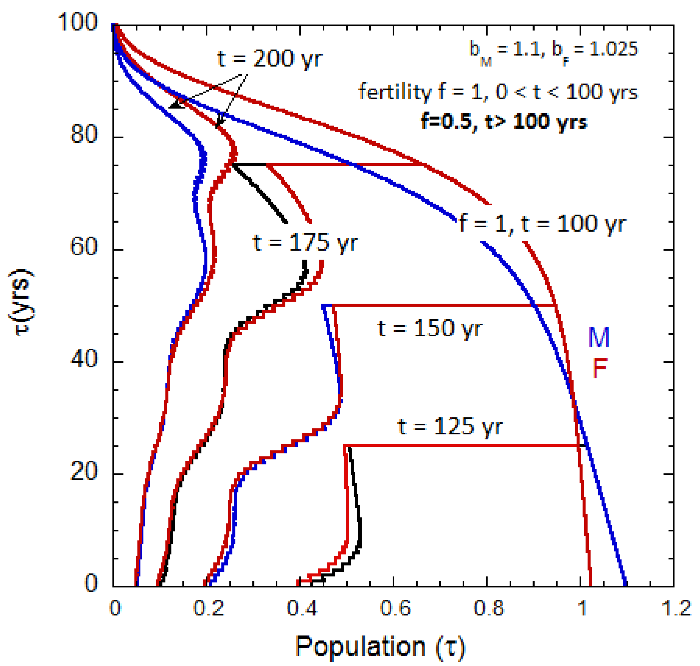


Figure 1. Evolution of the population profile at epochs t = 125, 150, 175, and 200 years, during a fertility collapse with *f* = 0.5 when t > 100 years. The original profile at t = 100o years employs *f* =1 for the first 100 years of the simulation.

τ_{pk} = 25 yo and σ = 4 yrs. Different numerical approximations for the integration of the fertility function, ranging from a δ-function to different numbers of terms in approximations for the Gaussian, are examined in the Appendix. Late evolving populations are also considered there.

Roughly, each of the steps in the profiles of the low fertility scenario of Fig. 1 reflect a generation, and the profile at 100 years sees a society dominated by the elderly, a paligarchy (Gr. παλαιός, for old), or gerontocracy. Some of these profiles compare favorably with the population profiles of aging countries, e.g., Japan or Italy, as seen more clearly in Fig.2a.

Profiles for different values of *f* at *t* = 200 yrs are shown in Fig. 2. The population collapses when *f* < *1* (Fig2a), and waves of the collapse are felt through the generations. When *f* >1, the population increases, and the population explodes when f > 2, under which conditions the profile assumes the traditional form of the pyramid.

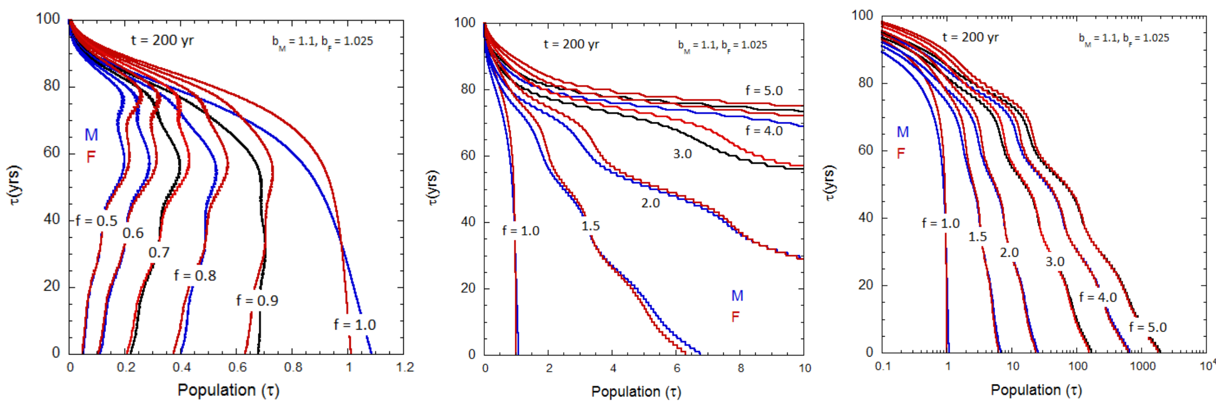


Figure 2a (left). Population profiles at t = 200 years for different values of *f*<1, as labeled. Figure 2b (center), profiles at *t* = 200 yrs for different values of *f*>1, as labeled, plotted on a linear population axis. Figure 2c (right). Same as Fig. 2b, but with the population plotted on a logarithmic scale.

Of some interest is the relationship of the fertility factors used here to the usual definition of total fertility rate. Here, an average woman who passes through her peak childbearing years has, on average, $f*(b_F + b_M)$ = 1 or (1.025 + 1.1) = 2.125 children with f = 1 in order to approach a steady state.[‡] This is the same value as the TFR required to have a steady population with time. In the simulation, all women abruptly start having $f*(b_F + b_M)$ as many children for 100 yrs, generating the population collapse and decline seen in Figs. 1 and 2a for f < 1, and the population growth and explosions seen in Figs.2b and 2c when *f* >1.

This sum is near the replacement value TFR = 2.1, but they are two different things. The TFR is obtained from knowing the average number of children per year born by women in all age ranges, and then calculating the number of children a hypothetical woman would have that year if a woman has proportional fertility throughout her lifespan. The present technique weights the fertility of the country according to the population of females in their child-bearing years.

[‡]Note that the use of 3 parameters to describe the fertility, f, b_F, b_M, *can be reduced to 2, namely the female fertility* f_{fem} = fb_F *and the sex ratio* ρ. *In the next section, this notation is adopted, with* $f_{fem} \rightarrow f$.

In future studies, it will be of interest to examine the relations between the fertility defined here and the TFR; the equation $dN/dt = f_0(f)N(t)$ in regards to evolving populations and how it compares to the simple equation $dN/dt = f_0N(t)$, $f_0 = const.$ for particle decay; the relationship of f to the population growth evolution as described by an exponential; and a comparison of profiles for different mortality laws, e.g,, as given in Table 1.

Ukraine Demographics

We examine the current demographics of Ukraine in order to make projections for the future of the Ukraine. The mortality, fertility and birth rates, and the 2023 population profile of Ukraine are treated in order.

Mortality

The mortality rates from 1980 – 2016 are remarkably similar and smoothly behaved, as can be seen with special clarity from the animation of Ukrainian M and F mortalities.[9] Here we provide stills of the Ukrainian age-stratified male and female mortality rates derived from the 5-year data for epochs 2002-2007, 2013-18, and 2018-2023. Fig. 3(a) shows the profile for the 2002-2007 epoch. It is remarkable in its smoothness, yet these vanilla profiles were common from 1980 to 2016. Note also that the mortality rates of 90-100 yo men and women are virtually identical from 1980-2008.

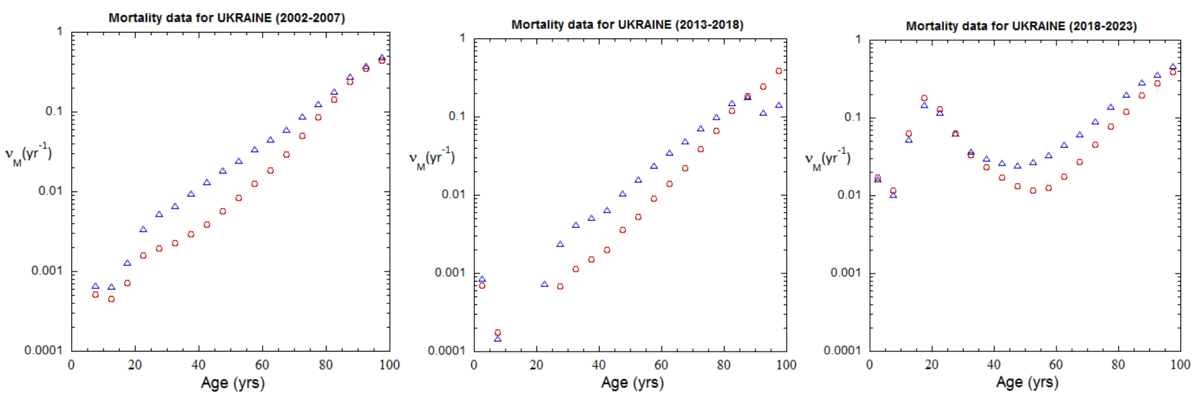


Figure 3. Ukrainian M and F mortality rates in epochs (a, left) 2002-2007, (b, center) 2013-2018, and (c, right) 2018-2023.

An anomaly in the rates is an odd, puzzling dip in 90-100 yo male mortality from 2010-2015, though it shows up as early as 2007 and persists until 2016 (keeping in mind the 5-yr smoothing of the rates). The female mortality rates remains effectively constant at all old (>60 yo) ages during the entire period 1980-2020, but the male mortality of 90-somethings becomes unusually low during the period from 2010 to 2015.

The dip of the male mortality of 90 -100 yo men below comparably aged females from 2010-2015 is hard to understand, requiring perhaps the male nonagenarians to become unusually healthy during this time period, or there to be an addition of male nonagenarians by immigration, which are both absurd. The center panel, Fig. 3(b), shows the dip at its extreme. It is the first instance out of 8 countries I've inspected that show male mortality less than female mortality.

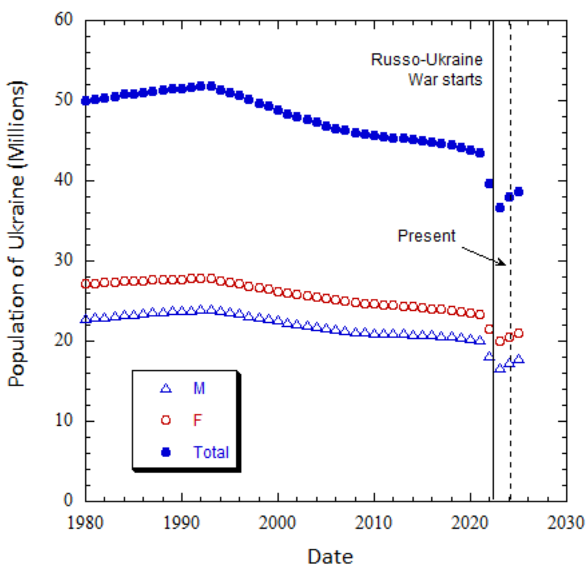
We don't understand why, in general, female mortality is less than male mortality, so it might be even harder to understand deviation from this behavior.

The tragic peak in the mortality of the young resulting from a combination of death and migration is seen in Fig. 3c in consequence of the Russia-Ukraine War. This mortality rate is derived from the epoch 2018-2023, when the war was raging. Similar mortality-rate plots are seen for epochs 2017-2022 and 2019-2024.

Fertility and Births

The male, female, and total populations of Ukraine from 1980 through the current epoch are plotted in Fig. 4a. The peak population of 51.8 million people was reached in 1992, and the population of Ukraine has been declining ever since.

Here it is important to note that to take a census of a country, the statisticians and demographers have to determine who is counted in the census, for example, citizen versus temporary or permanent residents, and the boundaries of the country as determined by national sovereignty or popular acclamation. The population of Crimea was just under 2 million people in 2014, compared to a Ukrainian population of 45.3 million, so this would have appeared as a sizable increase in the mortality rates. To the extent that there is no inflection in the Ukrainian population when Crimea was annexed by Russia in February/March, 2014, the analysis giving the UN data for Ukraine must have a peculiar way to tally the Ukrainian population. Note the optimistic UN projections for increasing Ukraine population later than 2023.



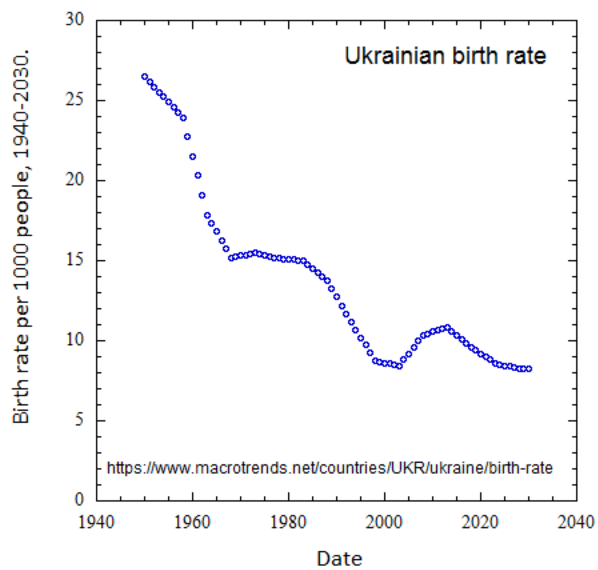


Figure 4a. (left) Male, female, and total population of Ukraine from 1980 to the present, as well as projected populations from 2023-2025, from UN data. Figure 4b. (right) Ukrainian birth rate per 1000 people.[6]

Fig. 4b shows the Ukrainian birth rate, spanning the years 1950 to 2030. The birth rate is defined in terms of number of births per 1000 people, so has a complex relation to female fertility, which depends on the age structure of the populations. Variations in birth rates can therefore result from variations in evolving population profiles, while the female fertility remains constant.

Figure 5 gives a differential picture of the population data in terms of year-over-year percentage changes (also referred to as annual percentage increase, or APC). The left panel shows the fine detail of population changes from 1980-2020. Fertility seems not to have been impacted by Chernobyl but fell off a cliff around 1991 at the time of the demise of the Soviet Union, which accounts for the population peaking in 1992. Some increases in fertility, though never reaching replacement levels, occurred from 2000-2010, but fertility has been declining since then through the current epoch.

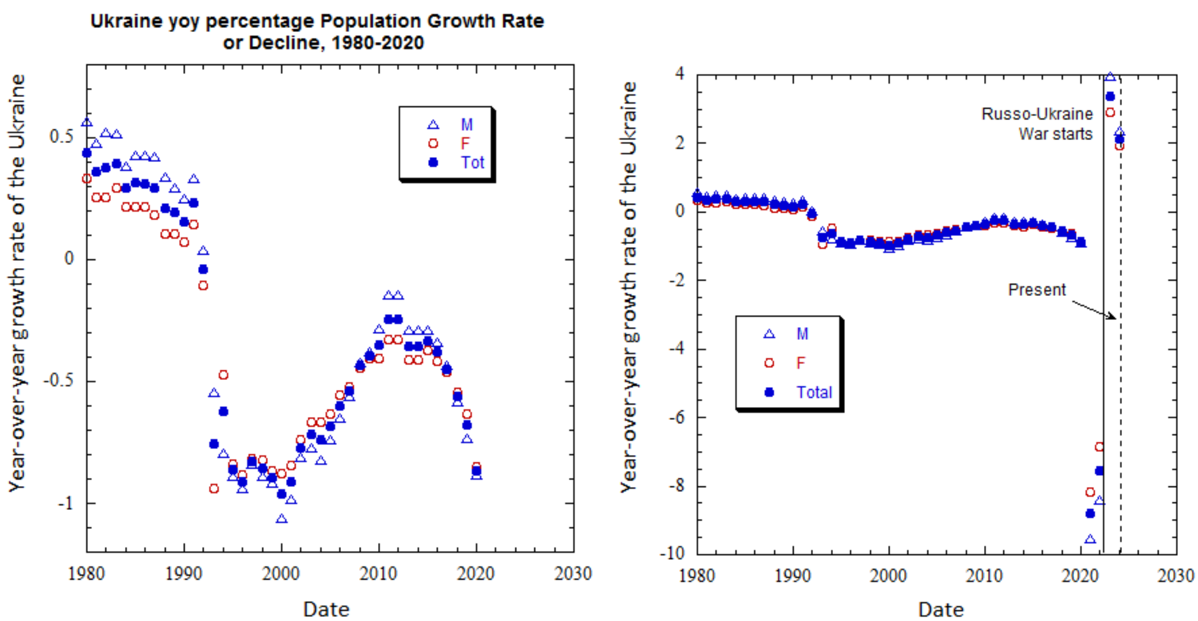


Figure 5. The year-over-year percentage rate of population increase or decrease, or APC, for Ukraine from 1980-2020 (left panel), and from 1980-2024 (right panel; note change of scale).

The right panel of Figure 5 shows the impact of the Russia-Ukraine War. The decline in population by some 15% in the first 2 years of the war agrees with estimates from the Ukrainian 2023 population profile described below. Results later than 2022 increasingly rely on extrapolations into the future. The scenario painted by the UN demographers is apparently of a recovering Ukraine following the war; thus, the large positive APC, after which the population follows a monotonic decline, apparently assuming that fertility rates of a future Ukraine follow the rates of the pre-war era.

I make independent projections in the next section to compare with the UN predictions.

The UN data for Ukraine[1] can also be used to calculate sex ratios and youth birth functions from 1980 to the present. The sex ratios of males to females at birth are shown by the X symbols, and are scaled by the left axis in Fig. 6. This is calculated from the ratio of populations of boys to girls between 0 and 4 yo. The right axis denotes the youth birth fraction, which is defined as the ratio of the population of women within a given age range of reproductive fertility to the total birth rate. The available data is in 5-yr age cohorts, so populations of young women in 5-yr age ranges are used in the first approximation.

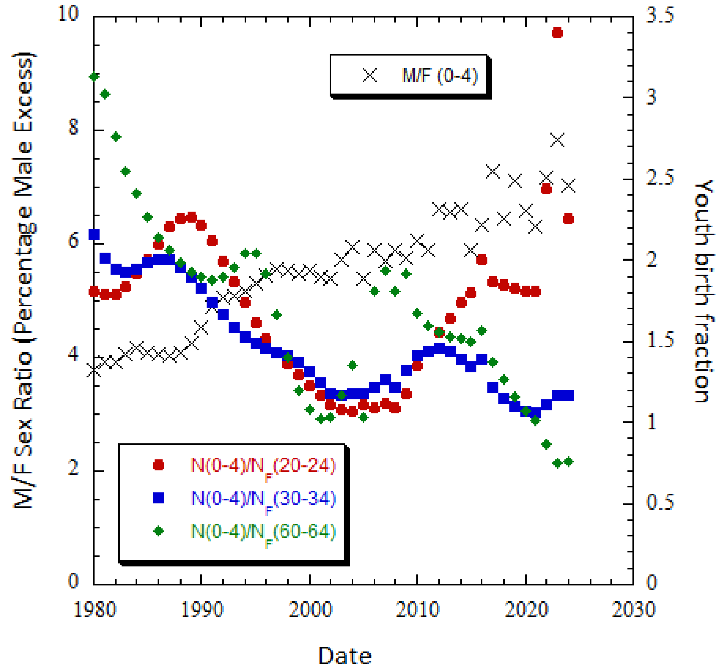


Figure 6. Sex ratio percentages ρ are given by the symbols marked by X and left axis. Ukrainian youth birth fractions for women with ages of 20-24 (circles), 30-34 yo (squares), and 60-64 yo (diamonds) are given by the filled symbols and right axis.

The Ukrainian youth birth fractions of women in the 20-24 yo, 30-34 yo age ranges and, for comparison, the 60-64 yo age range, to the total number of children in the 0-4 year range are shown in Fig. 6. The total number of children in the 0-4 age range is taken as a proxy for the birth rate throughout the country.

What the youth birth fraction essentially gives is the number of births per women in a specified age range to produce the contemporaneous birth rate. When there are few women in an age range, then they musth ave to have more babies to account for the births. As can be seen, the youth birth fractions for the 20-24 yo and the 30-34 yo ages are better behaved and give more reasonable values than the birth fractions for the 60-64 yo women, as expected. The Ukrainian youth birth fractions are in the range of 1-2 babies per young woman, that is, a fertility factor *f* between 0.5 and 1.

Following the start of the Russia-Ukraine war, the youth birth fraction would have to reach very large values if the births were primarily from 20-24 yo women, because these young women have almost all left (see next section). The birth correlates best with the population of 30-34 yo women, as seems reasonable, given common intuition about human nature.

Ukrainian 2023 Population Profile

The 2023 population profile using 1-yr UN data is found on Wikipedia.[8] They are in accord with the 5-yr data from PopulationPyramid.net,[1] as can be seen from Fig. 7.

A gash in population profiles reflects tragic circumstances, whether the population loss is due to the war or to emigration. A naïve calculation of the number of "missing" Ukrainian youth is obtained by assuming that the pre-war population profile, evolved to 2023, is given by the curved beaded data.

The area demarcated by this assumed population and the UN 2023 age-stratified populations in the 14-37 yo age range represents a population deficit of $1.96\mathrm{x}10^6$ males and $2.08\mathrm{x}10^6$ females, or about 4 million people in total. The population deficit for Ukraine from 2021-2023 UN data is about 7 million (Fig. 4), indicating considerable additional losses from older and younger persons. These huge losses explain the large negative rates in Fig. 5b for 2021 and 2022. The later large positive rates are UN projections and seem to require significant recovery of the Ukrainian population to pre-war levels.

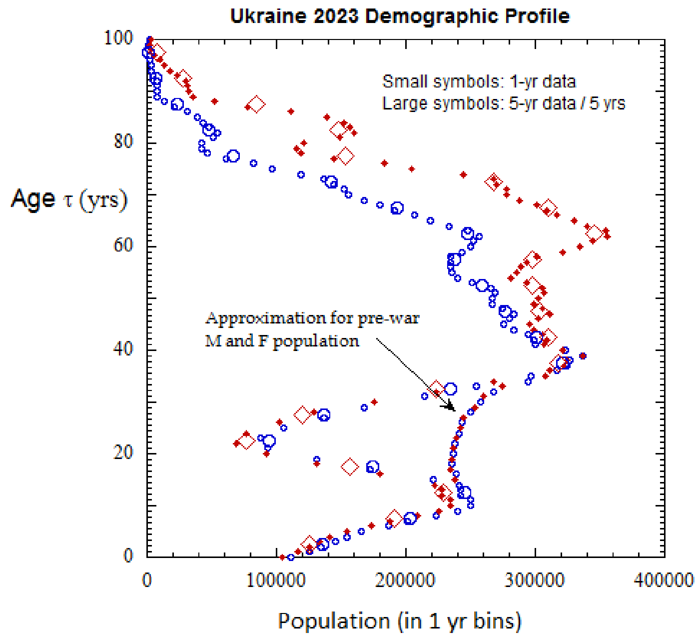


Figure 7. Ukraine population profile for 2023, showing 1-yr data (small symbols) and 5-yr data (large symbols). The beaded data gives an approximation for the pre-war population evolved to 2023, as if there had been no war. The enclosed area demarcates a population deficit of $2.0\mathrm{x}10^6$ males and $2.1\mathrm{x}10^6$ females between 14 and 37 years of age.

Two scenarios will be considered. In Scenario A, M and F population profiles are evolved into the future starting from 2023 UN data for Ukraine shown in Fig. 7. The same goes for Scenario B, except it uses the approximation for the pre-war M and F populations evolved to 2023 for persons between 14 and 37 years of age, also shown in Fig. 7.

Ukraine Population Projections

All the tools are in our hands to chart the projected evolving population profiles and total population of Ukraine under a range of assumed scenarios.

The standard scenario A follows a Ukrainian population evolving from the 2023 profile,[8] with a mortality law following the behavior of epoch 2002-2007 shown in Fig. 3a. In the projection, the birth rates are determined by the population of 30 yo Ukrainian women, so the code essentially uses a δ-function for female fertility at $\tau_{pk} = 30$. Fig. 6 suggests fertilities f in the range 0.5-1.0, so for the middle Scenario A, we take f =0.75, and a sex ratio $\rho = 1.06$ is assumed, as suggested by Fig. 6.

Fig. 8 shows the evolving profiles for two epochs, the years 2023 and 2063. An animation of this scenario gives a better view of the results.[9] Besides the aging population waves, what's notable is that the structure in the profiles remains as the profile evolves, because the code essentially uses a δ-function approximation to the fertility (the Appendix shows related behavior, where the δ-function retains discrete structure in the test profile evolution). Using a Gaussian model with τ_{pk} = 30 will give an improved projection and a somewhat smoother profile, but precise modeling requires comparison with birth mother data, and the overall picture is not likely to change much.

Just by looking at Fig. 8, one can see that the projected Ukrainian population is rapidly declining over the next 40 years. A simple integration, shown in Fig. 9, gives the projection for the evolving Ukrainian population for middle Scenario A, *f* = 0.75, from 2023 to 2070, to which we add the earlier 1980-2023 population data from Fig. 4a. The APC. or year-over-year percentage increase, is also shown. Note the temporal structure in the projected APC arising from the structure in the initial 2023 Ukraine population profile and its evolution through aging, dying, and giving birth.

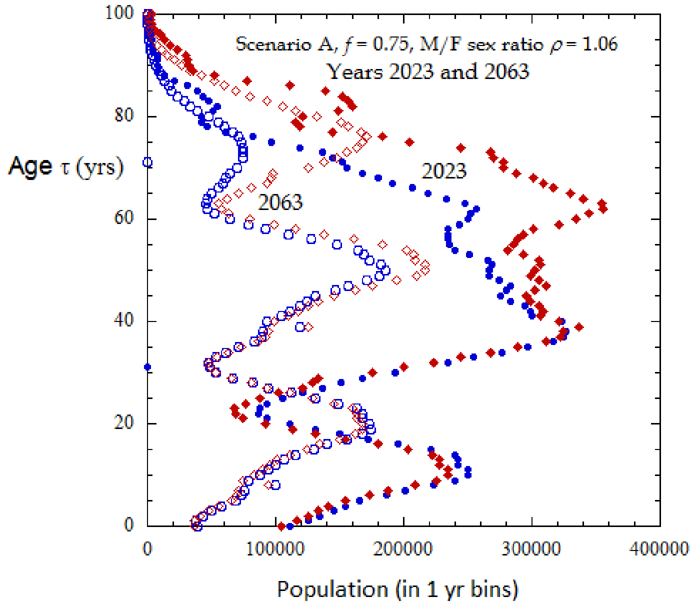


Figure 8. M and F population profiles for Ukraine in 2023 ((solid sybols) evolved to the year 2063 (open symbols)for scenario A with *f* = 0.75.

UN data for the M, F, and total Ukrainian populations from 1950 through the current year are available, though of course anything beyond the present (1/2024) is hypothetical. Fig. 9 shows the UN Ukrainian population from 1980 to the year 2024. Remarkably, the demographers predict a rebound of the Ukrainian population in 2024-2025 by 2.0x10^6 people, after declining by 6.79x10^6 from 2019-2023.This odd forecast accounts for the reversal of the populations from 2023-2025 and the positive spike in the APC from 2023-2025.

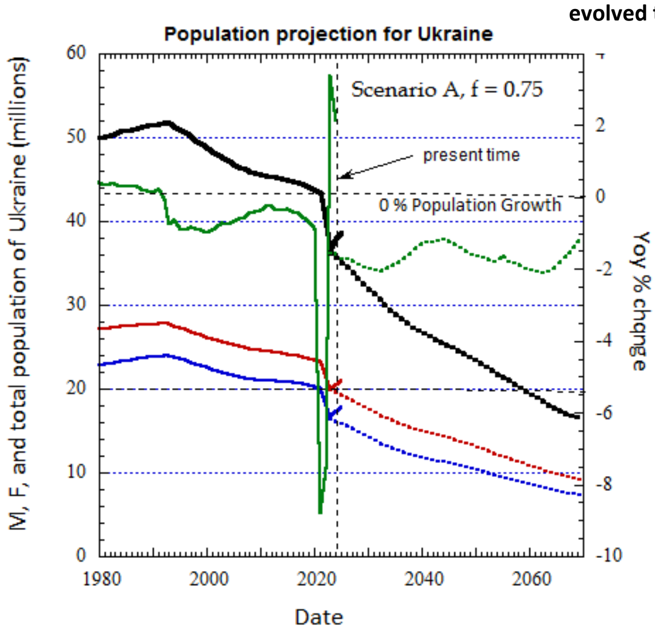


Figure 9. M, F, and total populations of Ukraine, including projections from 2023 to 2070, for Scenario A, *f* = 0.75. Also shown on the right axis and by the green lines is the annual (total population) percentage change (APC).

Consequently, we consider an alternative scenario B, using the initial 2023 population profile with the approximation for the pre-war 2019 male and female population evolved to 2023, as shown in Fig. 7. As noted earlier, this represents a postwar recovery of 4.1x10^6 people, so is even more optimistic than the UN demographers.

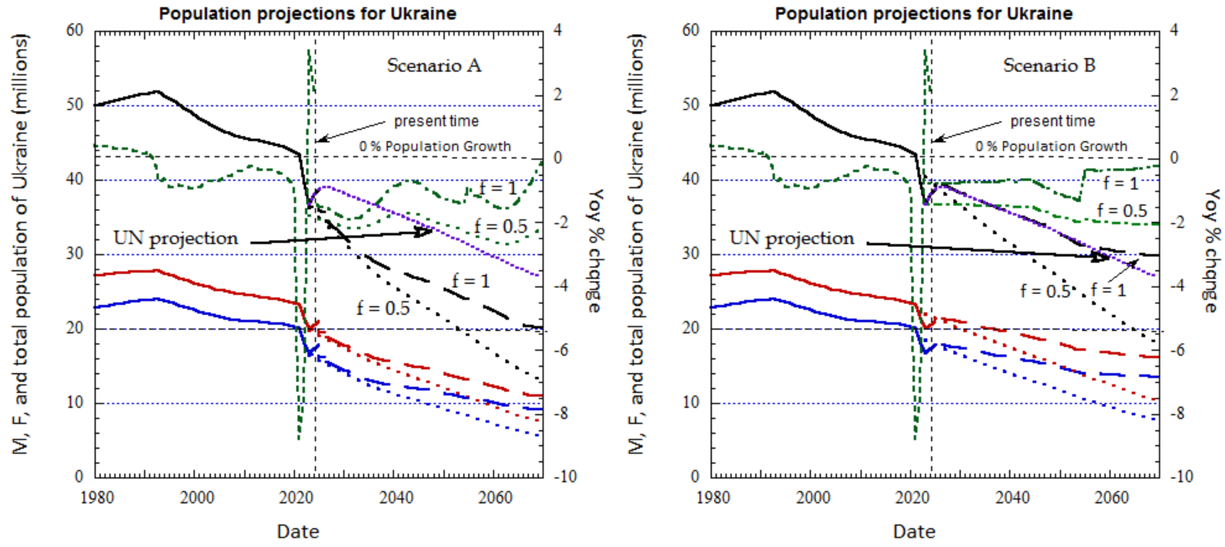


Figure 10 (a) Ukraine population projection for Scenario A, where the 2023 Ukraine population profile (Fig. 7) is evolved forward in time, for fertility factors *f* = 0.5 and *f* = 1. Male, female, and total populations are shown by the blue, red, and black curves, respectively. The purple dotted curve is the UN projection. The green curves are the APC given by the scale on the right. (b) Ukraine population projections for Scenario B, where the young, 2023 Ukraine population is assumed to fully recover to a population that approximates the 2019 Ukrainian population evolved to 2023. Populations and APC for for fertility factors *f* – 0.5 and 1 are shown.

Fig.10a shows results of calculations for scenario A, with *f* = 0.5 and *f* =1.0. Fig. 10b shows results of calculations with scenario B, also with *f* = 0.5 and *f* = 1.0. As can be seen, the projected Ukrainian population declines to some 2.0x10^7 people by 2070 in the optimistic *f* = 1.0 case for scenario A, far below the UN projection. In contrast, the UN demographic projection is in accord with scenario B when $f \approx 0.9$.

The projected 2070 Ukraine populations from the UN[1] and the predictions for scenarios A and B are listed in Table 3. Scenario A shows a population of Ukraine in 2070 to be, in the best case, 40% of what it was at its peak in 1992. Even the absurdly optimistic scenario B shows a reduction in Ukrainian population from its peak by a factor of nearly 2.

Table 2. Predicted Ukrainian population (millions)

	Scenario A	***Scenario B***	***UN***
f = 0.5	***13.1***	***17.9***	
f = 0.75	***16.4***	***23.3***	
f = 1.0	***20.2***	***29.6***	
UN			***26.9***

Discussion

A theory of population profiles has been developed and applied to Ukraine demographic data. The availability of population data from the UN[1] allows independent analyses, such as this study, which depends on the reliability of the data, always an open question.

The main results of the study for Ukraine are presented in Fig. 10 and Table 2 for Scenarios A and B. Female fertilities are calculated for $f = 0.5$, 0.75 and 1 (corresponding to youth birth fractions of ≅ 1, 1.5 and 2, since $\rho \cong 1$), which bracket and bisect the fertilities of the Ukrainian female population from 1980 to the present (Fig. 6). The birth rate is assumed to be proportional to the population of 30 yo Ukrainian women multiplied by f.

In Scenario A, assuming fertilities of the past 40 years persist into the future, then the Ukrainian population collapses below 20 millions of people by 2070. One might ask about scenario B, which predicts that the Ukrainian population “only” declines below 30 million people by 2070. The UN projection gives, precisely, 26.9 million, so the current results are in accord with their number when $f = 0.9$.

But scenario B is extraordinarily optimistic and assumes that there will be a large-scale migration back to the war-torn country. More likely, those Ukrainian women who have sought refuge in EU countries will call for their elderly parents to join them. Furthermore, it requires uniformly high fertilities $f \approx 1$ to persist for the next 50 years, inconsistent with the historical record, and highly unlikely even during peacetime.

The strong correlation of birth rates with populations of young females, shown in Fig. 6, suggests a numerical experiment giving possibly disturbing ramifications for Ukraine: Do the correlations improve when considering the ratio of the product of the male and young female populations to the birth rate? If they do, that spells further doom for the Ukrainian population because of the horrendous losses of men from the war and migration.

If one subscribes to Malthus’s observation quoted at the beginning of this article, the profound deficit of young people and their low fertility leads to a bleak and unhappy future for the Ukrainian people, The question for the survival of a nation is, then, how to raise fertility rates.

Lest everything look dark for Ukraine, we note some causes for hope from the present tragic historical circumstances. Following national calamity, fertility has been reported to rise, as in the case following the black death in England around 1348. A possible explanation is that the deficit of people makes them more valuable, forces wages to rise, and gives workers more purchasing power and residual land for expansion and to raise children. Other examples are considered in both Malthus’s 1798 and much expanded1803 *Essay on the Principle of Population*.

Ukraine is rich in agricultural product, especially in the West, and has vast natural resources in the east. It is crossed north to south by the Dnieper River and borders the Black Sea. From this perspective, it has the potential to be a rich, prosperous and growing country. On the other hand, it is situated at a dangerous crossroads between east and west, and is beset by corruption. Until the war is resolved, the future of the Ukrainian nation is perilous.

Conclusions

There are infinitely many conceivable futures for a county. Population demographics can be used to predict the most probable future for a given scenario. In this study, I find that even should the Russia-Ukraine war end today, and there be some migration back to Ukraine, Ukraine's population will irreversibly decline to less than 30 million people within 50 years. If there is little return migration, which seems considerably more likely, the numbers could decline to less than 20 million Ukrainians within 50 years.

It is hard not to be pessimistic, given events over the past several years, and these calculations generally confirm that pessimism. Yet dynamic countries with less than 20 million people exist: e.g., Hungary and Israel, each with about 10 million people. Yet Hungary is also facing population decline, and Israel depends on the orthodox and the Jewish diaspora making aliyah to sustain their population.

One might argue that immigrants to Ukraine, drawn by its fertile soil and mineral wealth, could grow and enrich the population. Similar arguments are made in the US context. Whether large-scale immigration aids or harms the host society is a topic that does not have a simple answer, but assuredly the culture changes in consequence.

Future events may make a mockery of these predictions and reveal them as pessimistic. Who knows? Ukrainians might suddenly start having lots of babies after the war, as happened in post-war USA. But unlike in the US, there can be no victory for Ukraine short of nuclear Armageddon, given that Putin sees the Russia/Ukraine conflict as existential. A large fraction of the young Ukrainian male population has been wiped out, and a large fraction of the young Ukrainian female population has sought asylum in the West, so the post-war situations of the US and Ukraine are hardly comparable. And a country turning to the West is not likely to find religion and increased fertility any easier than if it were to turn to Russia, which is also struggling with low fertility.

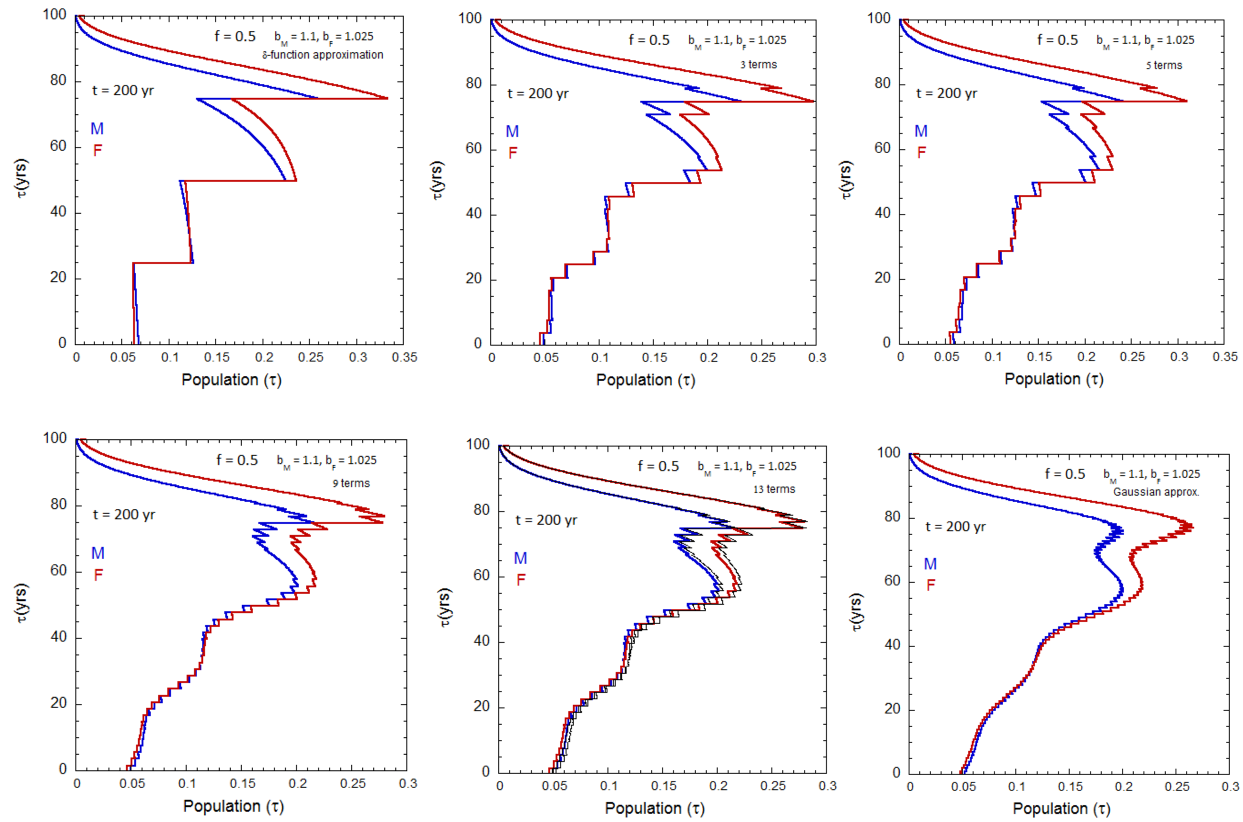
Returning to the Malthus quotation that triggered this investigation, if happiness is growth and fertility, and unhappiness is degeneration and mortality, then Ukraine likely faces a very unhappy future indeed.

This work is dedicated to the memory of Gonzalo Lira, a fearless journalist and bold partisan for free speech.

v.1 1/30/2024 v.2 3/28/2024 (minor typos)

Appendix: Profile Evolution for Different Approximations

It is of mainly academic interest to examine the shapes and appearances of evolving population profiles given different approximations to the fertility function.



Figures A1(a)-(e), all assume peak fertility age τ_{pk} = 25 yo, a fertility factor *f* = 1 for 0 < t < 100, f = 0.5 for 100 < t < 200, and birth ratios for males of b_M = 1.1 and females of b_F = 1.025. Population profiles are calculated at t = 200 yr. Fig. A1a, upper left, is a δ-function approximation at τ_{pk} = 25 yo. Figs. A1b -Abd, lower center, employ different numbers of terms in the Gaussian expansion, as labeled. The 9-term expansion is overlaid by faint black lines on the 13-term expansion. Fig. A1(e), lower right, is the full Gaussian calculation on an 0.1 yr grid. The mortality rates are the same as in the Numerical Experiment section.

As described in Fig. A1 caption, the panels show different approximations to the fertility function. The calculation uses the medium US male and female mortality rates given in the Numerical Experiment section, and a sex ratio at birth of ρ = 1.1/1.025 = 1.073. Fig. A1a, upper left, is the result δ-function approximation at τ_{pk}= 25 yr, eq. (6), with f_i(t) = 1 for 0 < t < 100, and f_i(t) = 0.5 for t >100. Five other panels, as labeled, use various numbers of terms for the expansion of the Gaussian in the approximation to the birth rate and fertility function, eq.(7). For example, the simplest 3 term expansion is 0.69*N_F(τ=25) + 0.137*(N_F(τ=21)+N_F(τ=29)). The 9 and 13 term Gaussians evaluate values of the female population at integer multiples of (σ/2) = 2 yrs.

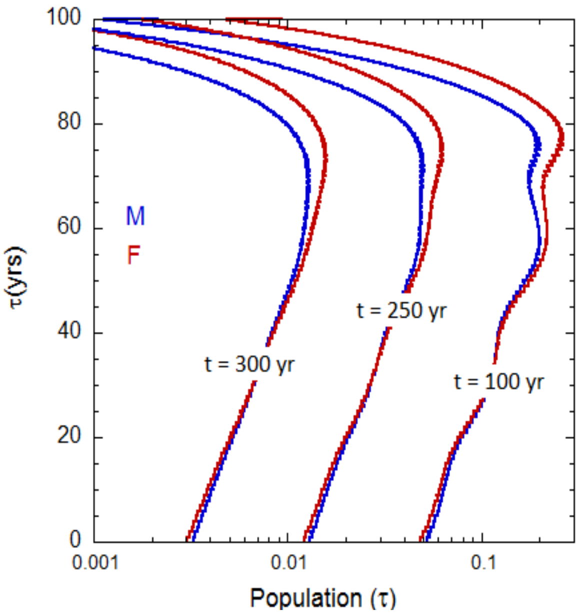


Figure A2. Gaussian calculations of the evolution of late-time population profiles at t=300, 250, and 200 yrs (typo in figure label). See text for parameters.

Of mathematical interest is the long-time evolution of population profiles for the prescribed, highly artificial conditions considered here. The Gaussian approximation gives the results shown in Fig. A2 using parameters above in the *f* = 0.5 scenario, though now extended to later times.

Most notable is that the populations at late times approaches the form of eq.(4), though with modified exponents reflecting the effects of *f* on population growth or decline.

$$\frac{N(\tau)}{N(\tau_0 \to 0)} = \exp\left[-\nu_i^f \tau - \left(\frac{\nu_0}{k_1}\right)\left(e^{k_1 \tau} - 1\right)\right]. \text{ (A1)}$$

Fig. A3 shows the δ-function approximation with the same parameters used in the calculation of Fig. A2. In this case, the structure is preserved and the population will never recover the form of Eq.(A1). The fertility function acts to smooth out the population profile with time for the artificial conditions assumed in the simulation. In reality, migration, famine, and war confound the conditions.

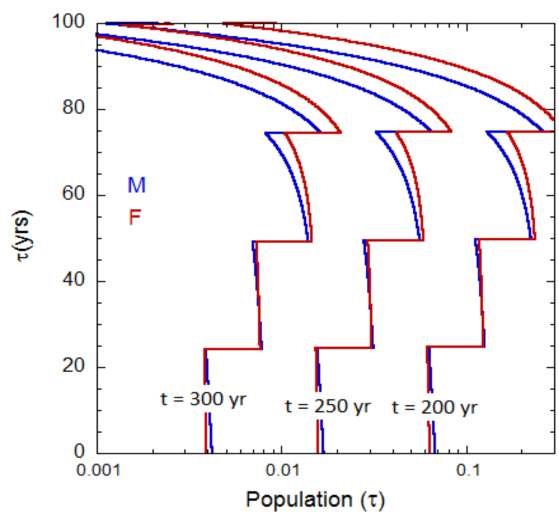


Figure A3. Same as in Figure A1, but using the δ-function approximation to female fertility, eq.(6).

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